THE STRENGTH OF THE EARTH'S CRUST

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PART II. REGIONAL DISTRIBUTION OF ISOSTATIC COMPENSATION

INTRODUCTION AND SUMMARY

The strength of the crust has been tested in the first part of this paper by those geologic changes which alter the surface of the earth, but not the density of its interior. If these changes in load initiate rather than merely coincide with vertical movements which serve to diminish the stress, they are thereby shown to be greater than the earth can permanently endure. If, on the other hand, the constructional forms persist, as in the two great deltas studied, then the movements which may exist in the crust due to those loads must be slower at least than the process of surface construction. Such loads consequently, unless counterbalanced by some factor not apparent, are within the limits of crustal strength.

But surface changes and the loads implied can be measured only in special cases. The previous attitude of the crust and the degree and direction of strain then existing in it are complicating factors which it is difficult quantitatively to evaluate. For these reasons the evidence yielded by geodetic investigation promises, in the end, more general and more accurate results.

It is an important conclusion, established by geodetic evidence, that the ocean basins are underlain by heavier matter than that beneath the continental platforms; the tendency through geologic time for the continents to rise relatively to the oceans may be correlated with this difference in density and the lightening of the land areas by the progressive erosion of the land surfaces. It is believed that the rejuvenative movements are in the direction of isostatic equilibrium. Fortunately for land-dwelling vertebrates, the crust is too weak for readjustment to be deferred until after the erosion of the lands, begun by the subaërial forces, shall have been completed by the sea.

But the power of geodetic research does not cease with the establishment of this cause of the maintenance of the differential relief between land surface and ocean floors. Beneath the surface of the continents it reveals heterogeneities of density and measures them against the more or less local relief above. To the extent to which areas of lighter or denser matter do not correspond to proportionately higher or lower relief, real strains either upward or downward are shown to exist through the crust. Over areas of plains which have not suffered much change for geologic ages, geodesy may thus reveal the existence of large crustal strain. On the contrary, in regions of mountainous relief, although the individual mountains are sustained by rigidity and bring local strains upon the supporting basement, geodetic study may show that there is close regional compensation of density balanced against relief, obliterating with depth the stress differences due to topography. These methods of research are thus capable of attacking the problem of the amount and direction of vertical strain existing in the crust under any part of the land surface and, to a lesser degree of accuracy, the crust beneath the sea. The breadth of the individual areas which depart from equilibrium in one direction may constitute also a vital part of the problem.

But although these are fields of research open to the geodesist, they are cultivated with much labor. The position of many stations on the surface of the earth must be determined by astronomic observations to within a fraction of a second of arc. Then a triangulation network, continent-wide, ties these together and shows

at each station, after allowing for the small errors of observation, what are the deflections of the vertical produced by the variations of relief and density. But this deflection for each station is the net result of all the relief from mean level and all the subsurface departures from the densities necessary to sustain that relief for distances of hundreds and, to a diminishing extent, even thousands of miles. The problem is made more soluble, however, by another and independent mode of attack. Observations on the intensity of gravity, when corrected for latitude, for elevation, for the surrounding relief and the density theoretically needed to sustain that relief, show the vertical component of those outstanding forces whose horizontal component was measured by astronomic determinations. It is seen that if the topography is known and its influence evaluated, and sufficient observations are reduced, the distribution of subcrustal densities and consequently the amount of crustal strains form soluble but complex problems.

The mathematical mode of investigation of such problems has, however, both its advantages and disadvantages. The advantages lie in giving quantitative results and in the test of the accuracy of the trial hypotheses by means of the method of least squares. A disadvantage lies in the necessity of erecting simple hypotheses in place of the complex realities of nature, in order to bring the data within the range of mathematical treatment. The precision of mathematical analysis is furthermore likely to obscure the lack of precision in the basal assumptions and through the apparent finality of its results tends to hide from sight other possibilities of the solution.

It is because of the geologic nature of the hypotheses on which the calculations concerning isostasy rest, and the geologic bearing of the results, that it is no act of presumption for the geologist to enter into this particular field of the geodesist.

The measurements of isostasy have been placed most fully on a quantitative basis by Hayford, and the science of geology is indebted to him in large measure. In the following consideration of the geodetic evidence attention will be confined almost entirely to his work, supplemented by that of Bowie. Hayford was the first to consider the influence of the topography and its compensation

to very great distances from each station, the first to make a considerable number of trial solutions upon various assumptions as to the depth of the zone of isostatic compensation, with the result that the reduction of the observations gave the dimensions of the earth with a considerably smaller probable error than any previous computations.[1]

But the conclusions in regard to the strength of the crust, drawn in the first part of this article from the study of deltas, stand in strong contrast to certain statements by Hayford and later by Hayford and Bowie. This second part must therefore outline the results reached by them and show what reconsiderations are necessary in order to bring into harmony their conclusions and the evidence derived from the previous geologic study. A preliminary review without criticism is given of their work in order to bring out their methods and results, and the geologic conclusions which they draw from those results. It is followed by a re-examination of the subject of regional versus local compensation. This is the problem of the size of the area over which, by virtue of the rigidity of the crust, irregularities of density and topography do not have individual relationships but do largely compensate each other over the region as a whole. It is a measure, therefore, of the areal limits of crustal strength. The tests employed by Hayford and Bowie are, as they note, indeterminate up to radii above 58.8 but less than 166.7 km. in length. Consequently Hayford did not change his opinion, based upon previous investigations, that regional compensation was limited to areas of less than one square degree. In

[1] The final publications have been issued by the United States Coast and Geodetic Survey and are as follows: Hayford, "The Figure of the Earth and Isostasy from Measurements in the United States (up to 1906)," 1909; referred to in this paper as Hayford, 1906; Hayford, "Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy," 1910; referred to in this paper as Hayford, 1909; Hayford and Bowie, "The Effect of Topography and Isostatic Compensation upon the Intensity of Gravity," 1912; referred to in this paper as Hayford and Bowie, 1912; Bowie, "Effect of Topography and Isostatic Compensation upon the Intensity of Gravity" (second paper), 1912; referred to in this paper as Bowie, 1912.

In addition Bowie has published in the *American Journal of Science*, "Some Relations between Gravity Anomalies and the Geologic Formations in the United States," (4) XXXIII (1912), 237–40.

The following discussion of their geodetic measurements and results will be confined to the work in these five papers.

this article, however, two other tests are applied which indicate that although in some areas compensation does not extend to 166.7 km. radius, in other areas it extends farther. It is concluded that the United States shows regional departures from isostasy over areas many times larger than Hayford thought to exist, as broad and in some regions probably somewhat broader than the areas of the Nile and Niger deltas, the breadth depending in considerable part upon the magnitude of the loads per unit of surface.[1]

GEODETIC MEASUREMENTS OF ISOSTASY BY HAYFORD AND BOWIE

Hayford's conclusions from deflections of the vertical.—The positions of many stations over the United States were determined with great accuracy by geodetic measurements from other stations, thus making a closed network. The positions were also determined by astronomic observation. The differences in latitude and longitude between the geodetic and astronomic positions give the observed deflections of the vertical due to the attraction of the surface irregularities and internal heterogeneities of the geoid. To account for these deflections the gravitative attraction upon the plumb-line at each station of all the topography from ocean bottoms to mountain tops within 4,126 km. was computed. The influence of the topography alone upon the direction of the vertical is known as the topographic deflection and averages a little over 30″. The average of the actually observed deflections are, however, but a fraction of this value. Consequently the excesses of volume represented by continents above oceans, and by plateaus on continents must be very largely balanced and neutralized by corresponding deficiencies of density in the crust beneath, which in turn explains how the larger relief is sustained. This is the theorem of isostasy. Various hypotheses in regard to the magnitude and distribution of these deficiencies in density under the continents, of excesses under the oceans, may be made, and the deflections recomputed on these successive suppositions and compared with the observed deflections.

[1] At the recent meeting of the Geological Society of America, December 30, 1913, to January 1, 1914, Professor W. H. Hobbs gave a paper on "A Criticism of the Hayfordian Conception of Isostasy Regarded from the Standpoint of Geology." The writer did not have the pleasure of hearing this paper, but it is clear that Professor Hobbs has attacked independently the same problems as here discussed.

The difference is the residual error due to the partial incorrectness of a hypothesis. The exactly correct hypothesis would reduce all residual errors to zero except for the errors of observation and computation. A hypothesis which approximates to the truth will give small residual errors. In a large mass of data the sum of the squares of the residuals as derived from different hypotheses serves as a test of the relative agreement of the hypotheses with nature, that hypothesis applying best for which the sum of the squares is a minimum. In all of the complete solutions a uniform distribution of compensation was assumed to exist from the surface to the bottom of the zone of isostatic compensation. That is, if the column under a certain portion of land was 3 per cent lighter than under a certain portion of water, then it was assumed that at any and every depth the two columns differed in density by 3 per cent. The differences abruptly terminate at the level where the two columns, the long but light land column and the short but heavy sea column, become of equal weight. At the level of this surface isostatic compensation is complete and there is hydrostatic equilibrium.

A tabulation of the probabilities of these hypotheses as applied to the whole of the United States is as follows:

TABLE III

Hypothesis	Sum of Squares of 765 Residuals
Solution B (extreme rigidity; depth of compensation infinite)	107,385
Solution E (depth of compensation 162.2 km.)	10,297
Solution H (depth of compensation 120.9 km.)	10,063
Solution G (depth of compensation 113.7 km.)	10,077
Solution A (depth of compensation zero)	18,889

The first investigation, that of 1906, favored Solution G, the final, that of 1909, as shown in this table, favored H. The most probable depth on the hypothesis of uniform compensation with depth and of equal depth of compensation for the whole United States was a little greater, being 122.2 km., 76 miles. It is seen, however, that there is but little change in the sum of the squares for a considerable range in the assumed depth. Further, Hayford states that the hypothesis of all compensation being attained in a 10-mile stratum whose bottom is at a depth of 35 miles is about as probable as the solution which he adopted.[1] Other variations in the hypothesis are also possible with about the same probable error.[2]

[1] 1906, p. 151.

[2] 1906, p. 153.

A distribution suggested by Chamberlin, of compensation greatest a little below the surface and diminishing to nothing at 178.6 miles, is also about as probable. Hayford therefore does not claim that his geodetic studies determine with precision the nature or depth of the distribution of compensation. The figure of 76 miles should therefore be used always with this reservation.

The residuals were classified into fourteen geographic groups. The most probable depths of compensation indicated for the several groups range from 66 to 305 km. According to Hayford, the evidence from these groups is, however, so weak and conflicting that he sees no indication that the depth of compensation is not constant over the whole area investigated.[1] He notes that, so far as the evidence goes, it indicates the depth of compensation to be greater in the eastern and central portions of the United States than in the western portion.[2] The subject is one which will be taken up later in the discussion of geodetic results.

In regard to the completeness of compensation, Hayford states:

> From the evidence it is safe to conclude that the isostatic compensation is so nearly complete on an average that the deflections of the vertical are thereby reduced to less than one-tenth of the mean values which they would have if no isostatic compensation existed. One may properly characterize the isostatic compensation as departing on an average less than one-tenth from completeness or perfection. The average elevation of the United States above mean sea-level being about 2,500 feet, this average departure of less than one-tenth part from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet (76 meters) thick on an average.[3]
>
> It is not intended to assert that every minute topographic feature, such, for example, as a hill covering a single square mile, is separately compensated. It is believed that the larger topographic features are compensated. It is an interesting and important problem for future study to determine the maximum size, in the horizontal sense, which a topographic feature may have and still not have beneath it an approximation to complete isostatic compensation. It is certain, from the results of this investigation, that the continent as a whole is closely compensated, and that areas as large as states are also compensated. It is the writer's belief that each area as large as one degree square is generally largely compensated. The writer predicts that future investigations will show that the maximum horizontal extent which a topographic feature may have and still escape compensation is between 1 square mile and 1 square degree. This prediction is based, in part, upon a consideration of the mechanics of the problem.[4]

[1] 1909, pp. 55–59.

[2] 1906, pp. 143, 146.

[3] 1909, p. 59.

[4] 1906, p. 169.

These conclusions imply a weakness of the crust surprising to the geologist and stand in marked contrast to those figures derived from the study of the deltas of the Nile and Niger. This subject also will be discussed later, as here it is desired to give only a summary statement of the methods and conclusions.

Hayford and Bowie on variations of gravity.—Regarding the relations of variations in gravity to isostasy, Hayford and Bowie state:

> As soon as it was evident that the proper recognition of isostasy in connection with computations of the figure and size of the earth from observed deflections of the vertical would produce a great increase in accuracy, it appeared to be very probable that a similar recognition of isostasy in connection with computations of the shape of the earth from observations of the intensity of gravity would produce a similar increase of accuracy. Logically the next step to be taken was therefore to introduce such a definite recognition of isostasy into gravity computations. Moreover, it appeared that if this step were taken it would furnish a proof of the existence of isostasy independent of the proof furnished by observed deflections of the vertical, and would therefore be of great value in supplementing the deflection investigations and in testing the conclusions drawn from them. In other words, the effects of isostasy upon the direction of gravity at various stations on the earth's surface having been studied, it then appeared to be almost equally important to investigate the effects of isostasy upon the intensity of gravity.[1]

In order to make the computations, the isostatic compensation was assumed to be complete under every topographic feature and uniformly distributed to a depth of 114 km. below sea-level, producing hydrostatic equilibrium at this depth. The mean density of 2.67 was taken as applying to the whole zone to this depth. Under land 3 km. high this gives a density of 2.60 from sea-level to a depth of 114 km.; under ocean 5 km. deep a density of 2.74 from ocean bottom to 114 km. below the bottom.[2]

The authors show that the topography and its compensation for the whole earth must be taken into consideration. On these assumptions the theoretic value of gravity was computed for every station, 124 in the final publication. This computed value is subtracted from the observed value and gives the "new-method" anomaly for each station. The results are shown in Fig. 5.

[1] Hayford and Bowie, 1912, p. 5.

[2] Hayford and Bowie, 1912, pp. 9, 10.

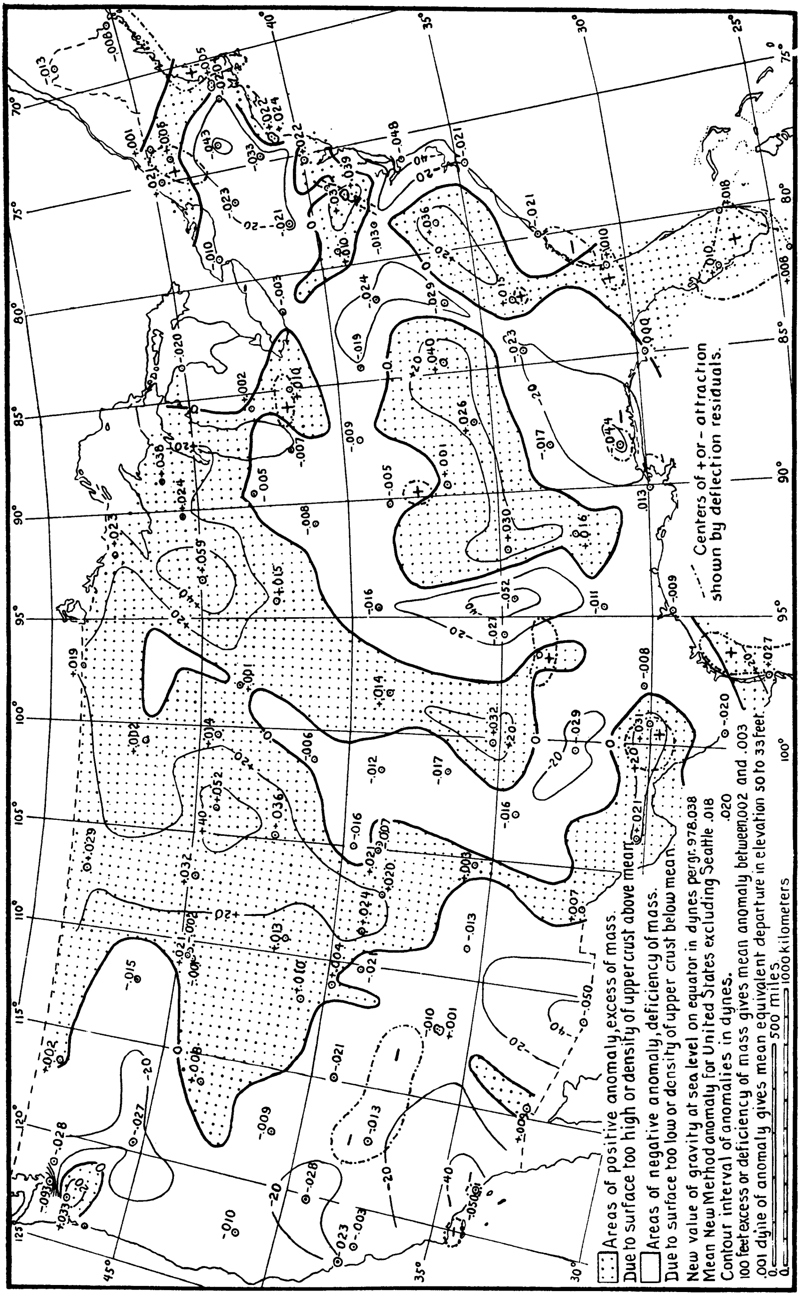

FIG. 5.—Lines of equal anomaly for new method of reduction, after Bowie

Of the two other principal methods of gravity reduction which have been previously used, the Bouguer reduction takes no account of isostatic compensation, postulating a high rigidity of the earth's crust, and neglects all curvature of the sea-level surface. The "free-air" reduction assumes that each piece of topography is compensated for at zero depth. These two reductions correspond thus to the limiting solutions tried for deflections of the vertical. The sum of the squares of the new method anomalies, when compared respectively with the similar sums derived from the hypothesis of rigidity and the hypothesis of compensation at depth zero, shows that the assumption of isostatic compensation uniformly distributed to a depth of 114 km. gives on the average smaller anomalies; is therefore much more probable and yields a more accurate value for the intensity of gravity. The mean anomaly of all stations in the United States without regard to sign, omitting the exceptionally large anomalies of the Seattle stations, is as follows:

New method	0.018 dyne[1]
Bouguer	0.063
Free air	0.028

The value of gravity for the United States Coast and Geodetic Survey office at Washington was determined as 980.112 dynes per gram. The mean new-method anomaly is consequently about 0.00002 of the value of gravity. The probable error of observation and computation is about 0.003 dyne. The errors may, however, frequently exceed 0.004 dyne and in rare cases may be as great as 0.010 dyne.[2] The fact that these measures of gravity are the forces acting on one gram will be understood through the rest of the paper.

Of the 124 stations, 32 have anomalies between 0.020 and 0.030, 12 have anomalies between 0.030 and 0.040.[3] Still smaller numbers of stations have higher anomalies. These anomalies measure departures in the earth's crust from the conditions of isostasy which were postulated. In the interpretation of the anomalies in terms of mass it is shown that a small excess of mass immediately below

[1] Bowie, 1912, p. 12.

[2] Hayford and Bowie, 1912, p. 79; Bowie, 1912, p. 13.

[3] Bowie, 1912, p. 13.

the station or a large excess at great depth or to one side may have the same effect. Therefore it is necessary to speak of the net effective excess or deficiency of mass.[1] A table is given showing these relations, and as a mean working hypothesis it is assumed that. ordinarily each 0.0030 dyne of anomaly is due to an excess or deficiency of mass equivalent to a stratum 100 ft. thick. In the final paper it is concluded:

> From the evidence given by deflections of the vertical the conclusion has been drawn that in the United States the average departure from complete compensation corresponds to excesses or deficiencies of mass represented by a stratum only 250 feet thick on an average. The gravity determinations indicate this average to be 630 feet instead of 250 feet. In neither case is the average value determined or defined with a high grade of accuracy. The difference between the two determinations of the average value is therefore of little importance. The determination given by the gravity observations is probably the more reliable of the two. Each determination is significant mainly as showing that the isostatic compensation is nearly perfect.
>
> The average elevation in the United States above mean sea level is about 2,500 feet. Therefore, from gravity observations alone the compensation may be considered to be about 75 per cent complete on an average for stations in the United States.[2]

This conclusion implies a somewhat greater rigidity to the crust than that which is stated for the deflections of the vertical, but in regard to the maximum horizontal extent which a topographic feature may have and still escape compensation the authors still express the belief that the limit is between one square mile and one square degree. "It appears from the inconclusive evidence furnished by the gravity observations that the radius of this area is probably less than 18.8 kilometers."[3]

This review of the work of Hayford on deflections of the vertical, and of Hayford and Bowie on the gravity anomalies has been given in order that the methods of the work, its bearings on the strength of the crust, and the conclusions which were reached, may be perceived. It is seen that a large difference of view as to the strength of the crust exists between this interpretation from the geodetic evidence and that from the geologic. In the following pages will be

[1] Hayford and Bowie, 1912, pp. 108–12; Bowie, 1912, p. 22.

[2] Bowie, 1912, pp. 22, 23.

[3] Hayford and Bowie, 1912, p. 102.

given a discussion which it is thought brings out certain errors in the conclusions drawn from the geodetic work and thereby reconciles the two lines of evidence.

REGIONAL VERSUS LOCAL DISTRIBUTION OF COMPENSATION

Conclusions on this topic by Hayford and Bowie.—Under this heading Hayford and Bowie state:

> The question whether each topographic feature is completely compensated for by a defect or excess of mass exactly equal in amount directly under it, or whether the topographic feature is compensated for by a defect or excess of mass distributed through a more extensive portion of the earth's crust than that which lies directly beneath it, is a very important one. The theory of local compensation postulates that the defect or excess of mass under any topographic feature is uniformly distributed in a column extending from the topographic feature to a depth of 113.7 kilometers below sea level. The theory of regional compensation postulates, on the other hand, that the individual topographic features are not compensated for locally, but that compensation does exist for regions of considerable area considered as a whole.
>
> In order to have local compensation there must be a lower effective rigidity in the earth's crust than under the theory of regional compensation only. In the latter case there must be sufficient rigidity in the earth's crust to support individual features, such as Pikes Peak, for instance, but not rigidity enough to support the topography covering large areas.
>
> Certain computations have been made to ascertain which is more nearly correct, the assumption of local compensation or the assumption of regional compensation only. In making such computations it is necessary to adopt limits for the areas within which compensation is to be considered complete. A reconnoissance showed that the distant topography and compensation need not be considered, for their effect would be practically the same for both kinds of distribution. As a result of this reconnoissance it was decided to make the test for three areas, the first extending from the station to the outer limit of zone K (18.8 kilometers), the second from the station to the outer limit of zone M (58.8 kilometers), and the third, to the outer limit of zone O (166.7 kilometers).[1]
>
> The average anomaly with regard to sign by the new method with local compensation, and the average anomaly by each of the three new-method reductions with regional distribution of the compensation are respectively −0.002, −0.001, −0.001, and −0.002 dyne. The means without regard to sign for the different distributions of the compensation are respectively, 0.020, 0.019, 0.019, and 0.020 dyne. These mean anomalies give only negative evidence.[2]

[1] Hayford and Bowie, 1912, p. 98. [2] Bowie. 1912, p. 22.

The problem may be tested in another way.

If local compensation be true, an unusually high mountain is underlain by unusually light matter and the intensity of gravity at a station on its top is less than if the mountain was supported by regional compensation and had matter of the mean regional density below it.

If the station is much below the average level of a mountainous region, local compensation implies, on the contrary, denser matter beneath and a higher value of gravity than would be given by regional compensation. These relations result in the following principle: For stations above the mean level, if local compensation be nearer the truth the hypothesis of regional compensation would tend to show its error by large negative anomalies. If regional compensation be nearer the truth, the hypothesis of local compensation would tend to show its error by giving large positive anomalies. For stations below the mean level the reverse would be true. But for any individual station other departures from the truth of that hypothesis of isostasy which gives the basis for the calculations may have greater influences and give larger anomalies than the question to be tested. Following this principle it is stated:

There are 22 stations in the United States in mountainous regions and below the general level and the means, with regard to sign, of the anomalies by the four methods of distribution are 0.000, +0.001, +0.003, and +0.005 dyne, while the means without regard to signs are respectively 0.017, 0.017, 0.018, and 0.019 dyne. For the 18 stations in the United States in mountainous regions and above the general level the means, with regard to sign, of the anomalies by the several methods of distribution of the compensation are +0.003, +0.003, 0.000, and −0.10 dyne. The means, without regard to sign, are respectively 0.018, 0.018, 0.017, and 0.020 dyne.

The mean, with regard to sign, of the anomalies for the stations at each of the two mountain groups, indicates that the theory of regional distribution of compensation to the outer limit of zone O, 166.7 kilometers is far from the truth. So far as may be judged from the other average anomalies no one method seems to have any decided advantage (see pp. 98–102 of *Special Publication No. 10*).[1]

Review and analysis of the evidence.—The present writer does not see in these computations any support for the hypothesis of local

[1] Bowie, 1912, p. 22.

compensation of the topography to between limits of one square mile and one square degree with the added suggestion of a radius less than 18.8 km., which has been advanced on other pages by the authors.[1] These figures merely show that, to the outer limit of zone M, radius 58.8 km., and probably to outer limit of zone N, radius 99 km., one method is as good as another for purposes of computation, which is not true in nature. The errors introduced by observation and computation, the errors introduced by the lack of recognition necessary in the preliminary hypothesis regarding the irregularities in the depth and distribution of compensation—these produce effects which overshadow the small systematic differences due to the hypotheses of local versus regional compensation. For the outer limit of zone O, radius of 166.7 km., a real distinction does, however, begin to appear in the data for the two groups of mountain stations. It is, however, very small and based upon a rather too limited number of stations to give quantitative reliability to the mean. Furthermore, as discussed in detail under a later heading, there is quite possibly a real difference between the limits of regional compensation and depth of compensation in the mountain regions of the West compared to other parts of the continent. Evidence drawn from the Cordillera cannot, therefore, be applied safely to the other portions of the United States.

Let the assumption be introduced that the limits of regional compensation are variable, ranging from 100 to 500 km. in radius. Such variable limits may well exist because of several factors; first, because of a real variability in the strength of the crust; second, because the greater vertical stresses could be carried only by smaller areas. In regions of mountainous relief due to folding, or of high anomalies due to great irregularities of density, the mean size of unit areas should therefore be less. On the whole the anomalies as well as the relief appear to be somewhat greater over the western United States. Third, in regions of recent block faulting or warping the stresses have presumably been lessened from what they were immediately before the movement. Such diminution of strain could take place by the breaking-up of a large unit area of crust into smaller units with differential movement among them, as

[1] Hayford and Bowie, 1912, p. 102.

well as by vertical movement of the whole area to a level best satisfying the stress. The western United States is known to be such a region, which in the late Tertiary and up to the present has been markedly affected by block faulting and differential vertical movements.

Suppose, then, that the mean radius of regional compensation in a mountainous region is 300 km. but that unit areas exist ranging in radius from 100 to 500 km. Of mountain stations located at random, a fraction of the total number would be situated within or near areas where regional compensation did not extend to 166.7 km. Let the stations be divided into one group consisting of those below the mean regional elevation and another group above the mean regional elevation. Let the anomalies be computed successively according to hypotheses of regional compensation to successive limits and the mean of the group for each limit be taken. This is the test applied by Hayford and Bowie. It has been seen that for radii of 18.8 and 58.8 km. the results are indeterminate. For a larger radius the group anomaly might be expected to show an increase as soon as the assumed radius exceeded the actual radii of a part of the areas. Consequently, if the hypothesis be true that the areas of regional compensation are variable in size, the mean anomalies of the two groups of 22 and 18 stations, found with regard to sign to be +0.005 and −0.010 respectively for radius of 166.7 km., do not show that regional compensation on the whole does not exist to those limits. It may indicate only that some areas are less than that radius. The mean radius of regional compensation may be 166.7 km. or possibly even larger. Other tests must therefore be sought which will give a more conclusive answer.

Further, it is to be noted that the mean anomalies with regard to sign for the hypothesis of regional compensation to radius of 166.7 km., although somewhat greater than for the other hypotheses, are yet of the same order of magnitude; and in all cases are but a fraction of the mean anomaly without regard to sign. Apparently, then, the assumption of regional compensation to 166.7 km. introduces a smaller error than the assumption of uniform and complete compensation with an average specific gravity of 2.67 to a constant depth of 114 km.

The test by adjacent stations at different elevations.—There is, however, another way of using the data given for stations situated well above and below the mean elevation of mountainous regions. If a pair of stations be taken close together, one far above the mean elevation, the other far below, they will presumably, because of their juxtaposition, be affected in much the same way by the errors incident to the hypothesis of uniform compensation through a depth of 114 km., with complete compensation at that depth. In order that good results may be obtained, however, the specific gravity of the local rocks should be carefully determined in order to have a correction for the mass between the stations. The parts of the anomalies due to the irregularities and incompleteness of compensation will ordinarily have the same sign and be of nearly the same value at the adjacent stations. This is indicated by the contour lines of Fig. 5, which show that in the same region the anomalies are of sufficiently regular gradation in magnitude to make the drawing of contour lines possible. The parts of the anomalies at the high and low stations due to errors in the hypothesis of local or regional compensation will, however, be of opposite sign. If, then, the algebraic difference of the anomalies for such a pair of stations be computed for successive hypotheses of broader regional compensation, the part of the anomalies due to *vertical* imperfection of the hypothesis will be largely eliminated. The algebraic difference measures the *horizontal* imperfection of the hypothesis. That hypothesis is favored whose assumed radius of regional compensation gives a minimum value to this algebraic difference. This test may be made by combining data given on p. 100, Hayford and Bowie, with p. 15, Bowie; although, because of incompleteness of the tables, this combination gives the data for only a few of the properly situated mountain stations. The best couple of stations for the application of this test consist of 42, Colorado Springs, and 43 Pikes Peak. Somewhat more distant stations—44, Denver, and 45, Gunnison—may also be added to the group. The tabulation is shown on p. 161 (Table IV).

It is seen that for three of the four Colorado stations the absolute value of the anomaly is least with regional compensation to 166.7 km. For the fourth station it remains practically constant for

all the cases. The anomalies were not computed for greater radii. The more convincing argument, however, for regional compensation to at least 166.7 km. radius in the vicinity of Pikes Peak is the fact that the *algebraic difference* of the anomalies between the top and bottom of the mountain, stations 43 and 42, is less than one-half for regional compensation to 166.7 km. radius than for the corresponding value given by the hypothesis of local compensation. The decrease in the difference is furthermore progressive with each

TABLE IV

Number and Name of Station	Elevation of Station in Meters	Distance from Mean Elevation in Meters within 100 Miles	Anomaly with Regional Compensation within Outer Limit of			
			Local Compensation. Radius 0.0 Km.	Zone K, Radius 18.8 Km.	Zone M, Radius 58.8 Km.	Zone O, Radius 166.7 Km.
COLORADO						
42. Colorado Springs......	1,841	−420	−0.009	−0.009	−0.010	−0.010
43. Pikes Peak.....	4,293	+2,035	+ .019	+ .011	+ .006	+ .002
44. Denver........	1,638	−574	− .018	− .016	− .009	− .001
45. Gunnison......	2,340	−380	+ .018	+ .021	+ .026	+ .016
Mean of 42, 44, and 45.........	....	−458	− .009	− .004	+ .007	+ .005
Algebraic Difference 43–42.......	....	....	+ .028	+ .020	+ .016	+ .012
43−(mean of 42, 44, 45)........	....	....	+ .028	+ .015	− .001	− .003
ARIZONA						
68. Yavapai.......	2,179	+512	− .001	− .001	− .001	− .009
69. Grand Canyon..	849	−824	− .012	− .011	− .011	− .021
Algebraic difference 68−69......	....	....	+0.011	+0.010	+0.010	+0.012

assumed widening of the zone. The result of adding the more distant stations, 44 and 45, favors regional compensation more markedly but is indeterminate between M and O. It would seem, then, that the front range of the Rocky Mountains in Colorado is upheld above the surrounding plains and parks by virtue of the rigidity of the earth.

The two stations in Arizona at 68 and 69 are well situated also to test the question of local versus regional compensation, but the

difference in the anomalies in this case is so nearly constant as to give an indeterminate answer. In the absence of more detailed statements by Hayford and Bowie the reason why the anomaly at the Grand Canyon station 69 reaches a larger *negative* value for regional compensation to 166.7 km. than for more limited compensation is not evident. The usual rule is that the progressive change in the anomaly for stations below the regional level for successive assumptions of wider regional compensation is by increments with a *plus* sign. Here, on the contrary, the change in the limits from zone M to zone O involves a *minus* increment of 0.010 in the anomaly. The cause of this reversal of sign, which the writer does not understand, seems in this case to be the cause of the indeterminate result.

Another line of evidence as to the effective limits over which the rigidity of the earth may extend is derived from a study of the grouping of the deflections of the vertical shown in illustrations 2, 3, 5, 6, Hayford, 1909, and the lines of equal anomaly for the new method of reduction, illustration No. 2, Bowie, 1912, the latter giving the basis for Fig. 5 of this article.

The test by areas of grouped residuals.—Illustration No. 5, Hayford, 1909, shows the grouping of the residuals of solution H for the north and south components of the deflections. An area with a plus sign corresponds to an excess of density to the south, or deficiency to the north. An area with a minus sign corresponds to a deficiency of density to the south, or excess to the north. A north-south chain of stations is therefore best for ascertaining the limits of the areas of north-south deflection of like sign. Such a belt extends across the United States between long. 97° and 98°, showing 9 areas covering 1,620 miles. The mean intercept is therefore 180 miles. This mean intercept must be somewhat less than the mean diameter.

Illustration No. 6, Hayford, 1909, shows the grouping of the residuals of solution H for the east and west components of the deflections. An area with a plus sign corresponds to an excess of density to the east, or deficiency to the west. An area with a minus sign corresponds to a deficiency of density to the east, or excess to the west. An east-west chain of stations is therefore best

for ascertaining the limits of the areas of like sign. Such a belt extends across the United States between lat. 38° and 39°.

The following adjustments in groups seem, however, fair to make, considering the lack of exact accuracy in any one station. At Cincinnati is a station showing small residuals opposite in sign to the stations on each side. If this is overlooked, three small groups become one of average size. In central Kansas a small minus area depending on a single observation may be likewise omitted. In western Colorado several small areas depending each upon two observations had their number diminished by one. The same was done in California. This gave 14 areas extending over 2,580 miles, a mean individual intercept of 184 miles. If 16 areas be taken, a mean value is derived of 161 miles. More weight, it is thought, is to be attached to the determination of 184 miles, and this is supported by the 180 miles shown by the north-south chain of stations.

The areas of like sign are between centers of excess and defect of mass. They are not, therefore, coincident with the areas of excess and defect, but in discussing the average size of areas, the one may be used as a measure of the other.

It may be concluded, therefore, that the deflections of the vertical show areas with departures from isostatic equilibrium in one direction and these areas average about 180 miles, 290 km., in mean intercept. The mean diameters of the areas of like sign are presumably somewhat greater. This would make the mean radius of areas of regional compensation, as indicated by similarity of sign among residuals, at least 166.7 km.—the radius of the outer limit of zone O used in the discussion of the gravity anomalies.

If we turn now to the anomalies shown by the determinations of gravity, Fig. 5, adapted from Bowie, shows their segregation into areas of like sign. The mean value without regard to sign for all stations excluding Seattle is 0.018 dyne per gram. Including the two Seattle stations the mean is 0.020 dyne. Between the contours for −0.020 and +0.020 lie tracts where the anomalies are within the mean limits. The areas of exceptionally large anomalies are above those limits. It is only these which form on this illustration well-defined inclosed areas, but even these are far from

regular in outline. The areas showing positive anomalies of more than 0.020 dyne were estimated roughly to average 130 by 240 miles across, a mean diameter of 175 miles. The areas showing negative anomalies of more than 0.020 dyne were found to average roughly about 190 by 250 miles, a mean diameter of 220 miles. The long narrow connections were neglected in making this estimate. Unit areas of more than mean anomaly may therefore be taken to average about 200 miles or 320 km. in diameter. The mean radius is therefore approximately that of the outer limits of zone O, 166.7 km.

The figures, although they correspond fairly closely to those derived from the deflections of the vertical, cannot in reality be very well compared, since these are areas selected because the anomaly rises above a certain magnitude; the others represent, on the contrary, a succession of contiguous areas between centers of excess and defect in mass without reference to magnitude. Apparently some influence blurs out the limitations of areas of small gravity anomaly. This will be discussed in a later part.

Now assume for the moment that isostatic compensation is uniform to the bottom of the zone, as postulated by the hypothesis; that is, that the residuals and anomalies are due to excesses or defects of mass which are uniformly distributed. Then, over any one area of excess or deficiency of mass, the deflections around it and anomalies within it signify a departure from compensation in one direction. This is a regional departure. If the strength of the crust was so small that it was able to support notable departures from compensation over areas of only one square degree or less, then these large unit areas could not exist. A vertical warping up or down would immediately take place until the broad region as a whole lay so close to complete compensation that its surface irregularities became subdivided into subordinate positive and negative areas of the limiting size. The sum of the excesses and defects of mass would approach zero in broad areas containing many unit departures. It would seem, therefore, that the geodetic results shown in Fig. 5, instead of indicating local compensation to limits of less than one square degree, show on the contrary a ready

capacity of the crust under the United States to carry over areas of from 5 to 10 or 15 square degrees, and exceptionally over even larger areas, departures from equilibrium greater than the mean. This agrees in order of areal magnitude with the Nile and Niger deltas. However, the influence of irregularity in the distribution of compensation with depth, and the magnitude of stress per unit area remain to be investigated.

[*To be continued*]